Question: What is the domain of

$$y = \sqrt{\frac{x+1}{x-3}} \ ?$$

Graph it.

domain: set of possible values of x that can be plugged into the function.

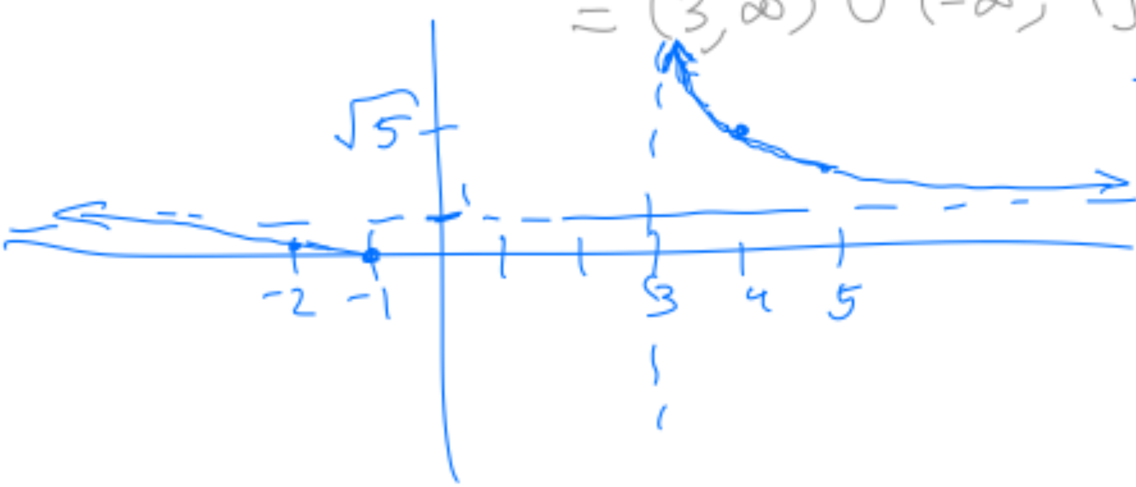
Issues: $\sqrt{\text{bubba}}$ ← need bubba ≥ 0.

$\dfrac{\text{howdy}}{\text{doody}}$ ← can't be zero.

$\Rightarrow \quad x \neq 3.$

$$\frac{x+1}{x-3} \geq 0 \longleftarrow \quad x > 3 \text{ or } x \leq -1$$

$$\text{domain} = \{x : x > 3 \text{ or } x \leq -1\}$$
$$= (3, \infty) \cup (-\infty, -1]$$



x	y
4	$\sqrt{\frac{5}{1}} = \sqrt{5}$
-1	0
-2	$\sqrt{\frac{-1}{-4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
5	$\sqrt{\frac{6}{2}} = \sqrt{3}$

as $x \to 3^+$, $y \to \infty$

$$\lim_{x\to 3^+} \sqrt{\frac{x+1}{x-3}} = \infty$$

$x \to 3^+$ means x is getting closer & closer to 3 but from the + side.

(3.1, 3.05, 3.007, 3.0002, ...)

$y \simeq \sqrt{\frac{\leftarrow \text{getting close to } 4}{\leftarrow \text{getting close to } 0^+}}$

y getting large & positive, without bound. ($\to +\infty$)

Notice that if x gets larger & larger ($x \to +\infty$), $y = \sqrt{\frac{x+1}{x-3}} \approx \sqrt{\frac{x}{x}} = 1$

$$\lim_{x\to +\infty} \sqrt{\frac{x+1}{x-3}} = 1$$

Another way to see it: we rewrite

dominant

$$\sqrt{\frac{x+1}{x-3}} = \sqrt{\frac{\frac{1}{x}(x+1)}{\frac{1}{x}(x-3)}} = \sqrt{\frac{1+\frac{1}{x}}{1-\frac{3}{x}}} \to 1^+.$$

getting tiny as $x \to +\infty$

What if $x \to -\infty$? $y \to 1^-$ (same argument)

Question: If $f(x) = x^3 + \frac{x}{2}$,

what is $f(x-2)$?

input

output

Answer: $f(x-2) = (x-2)^3 + \frac{x-2}{2}$

$x + (-2)$

$x^3(-2)^0$ $\quad$ $x^0(-2)^3$

$= x^3 + 3x^2(-2)^1 + 3x^1(-2)^2 + (-2)^3$

$+ \frac{x-2}{2}$

$= x^3 - 6x^2 + 12x - 8 + \frac{x}{2} - 1$

$= \boxed{x^3 - 6x^2 + \frac{25}{2}x - 9}$.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(A+B)^3$

$= A^3 + 3A^2B + 3AB^2 + B^3$

$A^3B^0 +$

$A = x$

$B = -2$

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③ Trig

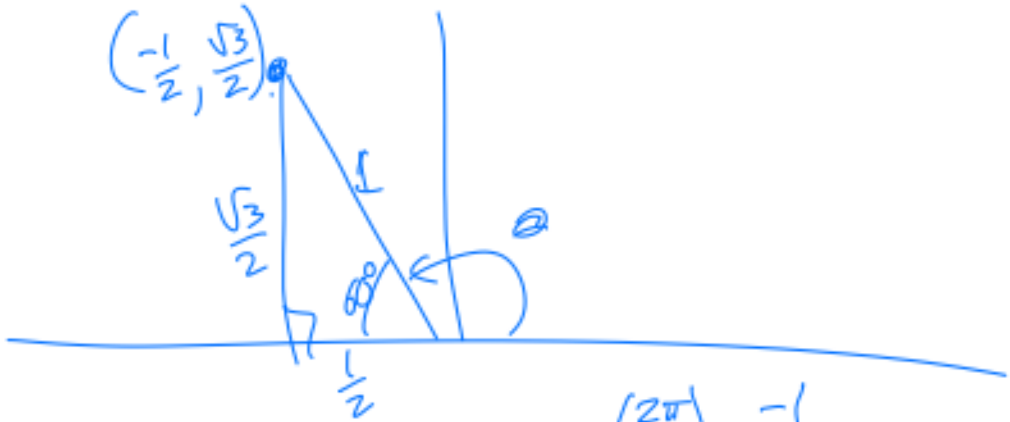
$\cos\left(\frac{2\pi}{3}\right) = ?$

$\sin\left(\frac{2\pi}{3}\right) = ?$

$a^2 + b^2 = c^2$

If $a = \frac{1}{2}$, $c = 1$

$\frac{1}{4} + b^2 = 1 \Rightarrow b^2 = \frac{3}{4} \Rightarrow b = \frac{\sqrt{3}}{2}$

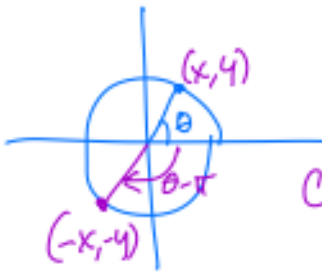


$x = \cos\theta =$
$y = \sin\theta$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Question: Simplify

$$F = \frac{\cos(\theta + A) + \cos(\theta - \pi)\cos(A)}{\sin\theta}$$



$\cos(\theta - \pi) = -\cos(\theta)$

$\cos(\theta + A) = \cos\theta\cos A - \sin\theta\sin A$
$\sin(\theta + A) = \sin\theta\cos A + \cos\theta\sin A$

$$F = \frac{\cancel{\cos\theta\cos(A)} - \sin\theta\sin A \cancel{+\cos\theta\cos A}}{\sin\theta}$$

$$= \boxed{-\sin A}$$

$1 - 2 - 1$
$1 + -2 + (-1)$

Back to limits

Example: find $\displaystyle\lim_{y\to 0} \frac{\sqrt{9y^4+2y^{16}}}{2y^3+6y^2}$.

Note

$\sqrt{A+B} \neq \sqrt{A}+\sqrt{B}$

$\sqrt{A\cdot B} = \sqrt{A}\cdot\sqrt{B}$

$\sqrt{A-B} \neq \sqrt{A}-\sqrt{B}$

$\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$

$(A+B)^C \neq A^C+B^C$

$(AB)^C = A^C B^C$

$\left(\frac{A}{B}\right)^C = \frac{A^C}{B^C}$

Rule of Thumb
Add + Subtraction Bad!
Mult & Division Good.

$$= \lim_{y\to 0} \frac{\sqrt{y^4(9+2y^{12})}}{2y^2(y+3)}$$

$$= \lim_{y\to 0} \frac{\sqrt{y^4}\,\sqrt{9+2y^{12}}}{2y^2(y+3)}$$

$$= \lim_{y\to 0} \frac{\cancel{y^2}\,\sqrt{9+2y^{12}}}{2\cancel{y^2}(y+3)} \qquad 2y^{12} \to 0^+ \text{ (fast)}, \quad y \to 0$$

$$= \frac{\sqrt{9}}{2\cdot 3} = \frac{3}{2\cdot 3} = \boxed{\frac{1}{2}}.$$

```
f(y)=9+2*y^12
show(f(y))
b=[f(y) for y in [-.3,.1,.07,-.0005]];
show(b)
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Evaluate

$2y^{12}+9$

[9.00000106288200, 9.00000000000200, 9.00000000000003, 9.00000000000000]

Example: $\displaystyle\lim_{y\to -1} \frac{\sqrt{9y^4+2y^{12}}}{2y^3+6y^2}$

same algebra

$$= \lim_{y \to -1} \frac{\sqrt{9 + 2y^{12}}}{2(y+3)} \qquad (-1)^{12} = 1, \quad y \to -1$$

$$= \frac{\sqrt{9+2}}{2(2)} = \boxed{\frac{\sqrt{11}}{4}}$$

Example:

$$\lim_{y \to \infty} \frac{\sqrt{9y^4 + 2y^{12}}}{2y^3 + 6y^2}$$

$$= \lim_{y \to \infty} \frac{\sqrt{9 + 2y^{12}}}{2(y+3)} \qquad \frac{\to \infty}{\to \infty}$$

dominant

$$= \lim_{y \to \infty} \frac{\sqrt{y^{12}\left(\frac{1}{y^{12}}\right)(9 + 2y^{12})}}{2y\frac{1}{y}(y+3)}$$

$$= \lim_{y \to \infty} \frac{\sqrt{y^{12}\left(\frac{9}{y^{12}} + 2\right)}}{2y\left(1 + \frac{3}{y}\right)}$$

$$= \lim_{y \to \infty} \frac{\sqrt{y^{12}}\sqrt{\frac{9}{y^{12}} + 2}}{2y\left(1 + \frac{3}{y}\right)}$$

$$= \lim_{y \to \infty} \frac{y^6 \sqrt{\frac{9}{y^{12}} + 2}}{2y\left(1 + \frac{3}{y}\right)}$$

$$\sqrt{y^{12}} = (y^{12})^{1/2} = y^6$$

$$= \lim_{y\to\infty} \frac{y^5\sqrt{\frac{9}{y^{12}}+2}}{2\left(1+\frac{3}{y}\right)}$$

∞, 0^+, 0^+

$$= \lim_{y\to\infty} \frac{y^5\sqrt{2}}{2\cdot 1}$$

∞

$$= +\infty.$$

Example

$$\lim_{x\to 0^+} \frac{e^{-1/x}-2}{x^2+4}$$